SOLUTIONS TO SELECTED QUESTIONS IN HOMEWORK 1

MATH 241

17.1.39

Proof. Let $z_1 = x_1 + y_1 i$, $z_2 = x_2 + y_2$, so $z_1 - z_2 = (x_1 - x_2) + (y_1 - y_2)i$, and $|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, which is exactly the distance between the points (x_1, y_1) and (x_2, y_2). □

17.2.9

Proof.

$$|\frac{3}{-1+i}| = \frac{|3|}{|-1+i|} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$
$$Arg(-1+i) = \frac{3\pi}{4}$$

so $\arg(\frac{3}{-1+i}) = 0 - \frac{3\pi}{4} + 2k\pi$, the principal argument takes $k = 0$ and get $Arg(\frac{3}{-1+i}) = -\frac{3\pi}{4}$. □

17.2.17

Proof.

$$3 - 3i = 3\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}))$$
$$5 + 5\sqrt{3}i = 10(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}))$$

Hence

$$(3 - 3i)(5 + 5\sqrt{3}i) = 30\sqrt{2}(\cos\frac{pi}{12} + i\sin\frac{\pi}{12})$$

□

17.2.30

Proof. $|-1+i| = \sqrt{2}$, $Arg(-1+i) = \frac{3\pi}{4}$, so the three roots are

$$2^{\frac{1}{6}}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}), 2^{\frac{1}{6}}(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}), 2^{\frac{1}{6}}(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12})$$

□

17.2.34

Proof. First solve $t^2 - 2t + 1 = 0$, then take the 4-th roots of it. □

17.3.4

Proof. $Im(z-i) = Im(z) - Im(i) = Im(z) - 1$, $Re(z+4-3i) = Re(z)+4$, so the graph is the line $y-1 = x+4$, or $y = x+5$ on the xy-plane. □

17.3.16

Proof. $Im(\frac{1}{z}) = -\frac{y}{x^2+y^2}$ if $z = x+yi$, so the set is $\{(x,y)|x^2+y^2+2y > 0\}$, which is the complement of a closed disk centered at $(0,-1)$ with radius 1. □